Appendix D

Spectral Resolution and Sensitivity Bandwidth in Spectrometers

For many of the spectrometers used at millimeter-wavelength observatories (filter bank spectrometers, correlation spectrometers, *etc*.), the true bandwidth of a single spectrometer channel is a convolution of the original gain response of the channel and any smoothing functions applied. This convolution of the response and smoothing functions affects two important factors:

$\Delta\nu_{FWHP} \equiv$ The full-width to half-power, or -3dB, bandwidth, which is the true "spectral resolution", and

$\Delta\nu_{SB} \equiv$ The "sensitivity bandwidth", which is the channel bandwidth used in calculations of the rms sensitivity in a spectrum.

The sensitivity bandwidth is defined as follows:

$$\Delta\nu_{SB} \equiv \frac{\left[\int_{-\infty}^{\infty} F(\nu)d\nu\right]^2}{\int_{-\infty}^{\infty}[F(\nu)]^2\,d\nu} \qquad \text{(D.1)}$$

Kraus (*Radio Astronomy,* 2nd Edition, page 7-8) defines a similar term.

Table D.1 describes the relationship between $\Delta\nu_{FWHP}$, $\Delta\nu_{SB}$, and the frequency sampling $\Delta\nu$ for various smoothing functions F(ν) assuming Nyquist sampling at $\Delta\nu$. At the end of this Appendix I show the calculation for each of these integrals.

For the filter bank spectrometers at the 12m, the channel response is a second order Chebyshev bandpass filter (which approximates a pill box function), there is no smoothing, and they are **not** Nyquist sampled. A single channel in the Millimeter Autocorrelator (MAC), on the other hand, has a sinc frequency response which is hanning smoothed. Since the convolution of a sinc with any function that is already band-limited withing the frequency response of the sinc leaves that function unchanged, we are left with a hanning function response. Therefore:

Table D.1: Spectral Resolution and Sensitivity Bandwidth

F(ν)	$\frac{\Delta\nu_{FWHP}}{\Delta\nu}$ [a]	$\frac{\Delta\nu_{SB}}{\Delta\nu}$ [a]
sinc	1.207	1.000
gaussian	2.000	1.505
hanning	2.000	2.667
hamming	1.820	2.935

[a] $\Delta\nu$ is the frequency sampling interval.

$$\Delta\nu^{FB}_{FWHP} = \Delta\nu \tag{D.2}$$

$$\Delta\nu^{HC}_{FWHP} = 2.000\Delta\nu \tag{D.3}$$

$$\Delta\nu^{FB}_{SB} = \Delta\nu \tag{D.4}$$

$$\Delta\nu^{HC}_{SB} = 2.667\Delta\nu \tag{D.5}$$

D.1 Function Integrals

D.1.1 Sinc

$$\begin{aligned}\Delta\nu_{SB} &= \frac{\left[\int_{-\infty}^{\infty}\frac{\sin\pi\nu}{\pi\nu}d\nu\right]^2}{\int_{-\infty}^{\infty}\left[\frac{\sin\pi\nu}{\pi\nu}\right]^2 d\nu} \\ &= \frac{\left[2\int_{0}^{\infty}\frac{\sin\pi\nu}{\pi\nu}d\nu\right]^2}{2\int_{0}^{\infty}\left[\frac{\sin\pi\nu}{\pi\nu}\right]^2 d\nu} \\ &= \frac{\left[\frac{2}{\pi}\left(\frac{\pi}{2}\right)\right]^2}{\frac{2}{\pi^2}\left(\frac{\pi^2}{2}\right)} \\ &= 1\end{aligned} \tag{D.6}$$

D.1.2 Gaussian

$$\begin{aligned}\Delta\nu_{SB} &= \frac{\left\{\int_{-\infty}^{\infty}\exp(-\ln(2)\nu^2)d\nu\right\}^2}{\int_{-\infty}^{\infty}[\exp(-\ln(2)\nu^2]d\nu} \\ &= \frac{\left\{2\int_{0}^{\infty}\exp(-\ln(2)\nu^2)d\nu\right\}^2}{2\int_{0}^{\infty}[\exp(-\ln(2)\nu^2]d\nu} \\ &= \frac{\frac{\pi}{\ln(2)}}{\sqrt{\frac{\pi}{2\ln(2)}}}\end{aligned} \tag{D.7}$$

$$= \sqrt{\frac{\pi}{2\ln(2)}}$$
$$\cong 1.50538$$

D.1.3 Hanning

The Hanning function is non-zero only from –(*N* – 1) to *N* – 1, where N is the number of channels which are being smoothed…

$$\Delta v_{SB} = \frac{\left\{ \int_{-(N-1)}^{N-1} \frac{1}{2}[1+\cos(2\pi v)]dv \right\}^2}{\int_{-(N-1)}^{N-1} \left\{ \frac{1}{2}[1+\cos(2\pi v)] \right\}^2 dv} \tag{D.8}$$

Normally, we take 3 channels and give them weights 0.25,0.5,0.25. Therefore, Δv_{SB} becomes

$$\Delta v_{SB} = \frac{\left\{ \int_{-2}^{2} \frac{1}{2}[1+\cos(2\pi v)]dv \right\}^2}{\int_{-2}^{2} \left\{ \frac{1}{2}[1+\cos(2\pi v)] \right\}^2 dv} \tag{D.9}$$
$$= \frac{\left\{ 2\int_{0}^{2} \frac{1}{2}[1+\cos(2\pi v)]dv \right\}^2}{2\int_{0}^{2} \left\{ \frac{1}{2}[1+\cos(2\pi v)] \right\}^2 dv}$$
$$= \frac{4}{\frac{3}{2}}$$
$$= \frac{8}{3}$$

D.1.4 Hamming

The Hamming function is just the Hanning function with different weighting.

$$\Delta v_{SB} = \frac{\left\{ \int_{-(N-1)}^{N-1} [0.54+0.46\cos(2\pi v)]dv \right\}^2}{\int_{-(N-1)}^{N-1} \{0.54+0.46\cos(2\pi v)\}^2 dv} \tag{D.10}$$
$$= \frac{\left\{ 2\int_{0}^{2} [0.54+0.46\cos(2\pi v)]dv \right\}^2}{2\int_{0}^{2} \{0.54+0.46\cos(2\pi v)\}^2 dv}$$

$$= \frac{(2.16)^2}{1.5896}$$
$$\cong 2.935078$$